CHAPTER 2
BLACKBODY RADIATION

2.1 *Introduction.*

This chapter briefly summarizes some of the formulas and theorems associated with blackbody radiation. A small point of style is that when the word "blackbody" is used as an adjective, it is usually written as a single unhyphenated word, as in "blackbody radiation"; whereas when "body" is used as a noun and "black" as an adjective, two separate words are used. Thus a black body emits blackbody radiation. The Sun radiates energy only very approximately like a black body. The radiation from the Sun is only very approximately blackbody radiation.

2.2 *Absorptance, and the Definition of a Black Body.*

If a body is irradiated with radiation of wavelength λ, and a fraction $a(\lambda)$ of that radiation is absorbed, the remainder being either reflected or transmitted, $a(\lambda)$ is called the *absorptance* at wavelength λ. Note that λ is written in parentheses, to mean "at wavelength λ", not as a subscript, which would mean "per unit wavelength interval". The fractions of the radiation reflected and transmitted are, respectively, the *reflectance* and the *transmittance*. The sum of the absorptance, reflectance and transmittance is unity, unless you can think of anything else that might happen to the radiation.

A body for which $a(\lambda) = 1$ for all wavelengths is a black body.

A body for which a has the same value for all wavelengths, but less than unity, is a grey body.

(Caution: We may meet the word "absorbance" later. It is not the same as absorptance.)

2.3 *Radiation within a cavity enclosure.*

Consider two cavities at the same temperature. We'll suppose that the two cavities can be connected by a "door" that can be opened or closed to allow or to deny the passage of radiation between the cavities. We'll suppose that the walls of one cavity are bright and shiny with an absorptance close to zero, and the walls of the other cavity are dull and black with an absorptance close to unity. We'll also suppose that, because of the difference in nature of the walls of the two cavities, the radiation density in one is greater than in the other. Let us open the door for a moment. Radiation will flow in both directions, but there will be a net flow of radiation from the high-radiation-density cavity to the low-radiation-density cavity. As a consequence, the temperature of one cavity will rise and the temperature of the other will fall. The (now) hotter cavity can then be used as a source and the (now) colder cavity can be used as a sink in order to operate a heat engine which can then do external work, such work, for example, to be used for repeatedly opening and closing the door separating the two cavities. We have thus constructed a perpetual motion machine that can continue to do work without the expenditure of energy.

From this absurdity, we can conclude that, despite the difference in nature of the walls of the two cavities (which were initially at the same temperature), the radiation densities within the two cavities must be equal. We deduce the important principle that the radiation density inside an enclosure is determined solely by the temperature and is independent of the nature of the walls of the enclosure.

2.4 *Kirchhoff's Law*

Kirchhoff's law, as well as his studies with Bunsen (who invented the Bunsen burner for the purpose) showing that every element has its characteristic spectrum, represents one of the most important achievements of mid-nineteenth century physics and chemistry. The principal results were published in 1859, the same year as Darwin's *The Origin of Species*, and it has been claimed that the publication of Kirchhoff's law was at least as influential in the advance of science as the Darwinian theory of evolution. It is therefore distressing that so few people can achieve the triple task of spelling his name, pronouncing it correctly, and properly stating his law. Kirchhoff and Bunsen laid the foundations of quantitative and qualitative spectroscopy.

Imagine an enclosure filled with radiation at some temperature such that the energy density per unit wavelength interval at wavelength λ is $u_\lambda(\lambda)$. Here I have used a subscript and parentheses, according to the convention described in section 1.3, but, to avoid excessive pedantry, I shall henceforth omit the parentheses and write just u_λ. Imagine that there is some object, a football, perhaps, levitating in the middle of the enclosure and consequently being irradiated from all sides. The irradiance, in fact, per unit wavelength interval, is given by equation 1.17.1

$$E_\lambda = u_\lambda c/4 \qquad 2.4.1$$

If the absorptance at wavelength λ is $a(\lambda)$, the body will absorb energy per unit area per unit wavelength interval at a rate $a(\lambda)E_\lambda$.

The body will become warm, and it will radiate energy. Let the rate at which it radiates energy per unit area per unit wavelength interval (i.e. the exitance) be M_λ. When the body and the enclosure have reached an equilibrium state, the rates of absorption and emission of radiant energy will be equal:

$$M_\lambda = a(\lambda)E_\lambda. \qquad 2.4.2$$

But E and u are related through equation 2.4.1, and u_λ is independent of the nature of the surface (of the walls of the enclosure or of any body within it), and so we see that the ratio of the exitance to the absorptance of any surface is independent of the nature of the surface. This is *Kirchhoff's Law*. (In popular parlance, "good emitters are good absorbers".) The ratio is a function only of temperature and wavelength. For a black body, the absorptance is unity, and the exitance is then the Planck function.

2.5 *An aperture as a black body*.

We consider an enclosure at some temperature and consequently filled with radiation of density u_λ per unit wavelength interval. The inside walls of the enclosure are being irradiated at a rate given by equation 2.4.1. Now pierce a small hole in the side of the enclosure. Radiation will now pour out of the enclosure at a rate per unit area that is equal to the rate at which the walls are being radiated from within. In other words the exitance of the radiation emanating from the hole is the same as the irradiance within. Now irradiate the hole from outside. The radiation will enter the hole, and very little of it will get out again; the smaller the hole, the more nearly will all of the energy directed at the hole fail to get out again. The hole therefore absorbs like a black body, and therefore, by Kirchhoff's law, it also radiates like a black body. Put another way, a black body will radiate in the same way as will a small hole pierced in the side of an enclosure. Sometimes, indeed, a warm box with a small hole in it is used to emulate blackbody radiation and thus to calibrate the sensitivity of a radio telescope.

2.6 *Planck's equation*

The importance of Planck's equation in the early birth of quantum theory is well known. Its theoretical derivation is dealt with in courses on statistical mechanics. In this section I merely give the relevant equations for reference.

Planck's equation can be given in various ways, and here I present four. All will be given in terms of exitance. The radiance is the exitance divided by π .(Equation 1.15.2.). The four forms are as follows, in which I have made use of equations 1.3.1 and the expression $h\nu = hc/\lambda$ for the energy of a single photon.

The rate of emission of energy per unit area per unit time (i.e. the exitance) per unit wavelength interval:

$$M_\lambda = \frac{C_1}{\lambda^5\left(e^{K_1/\lambda T} - 1\right)} \qquad 2.6.1$$

The rate of emission of photons per unit area per unit time per unit wavelength interval:

$$N_\lambda = \frac{C_2}{\lambda^4\left(e^{K_1/\lambda T} - 1\right)} \qquad 2.6.2$$

The rate of emission of energy per unit area per unit time (i.e. the exitance) per unit frequency interval:

$$M_\nu = \frac{C_3\nu^3}{e^{K_2\nu/T} - 1} \qquad 2.6.3$$

The rate of emission of photons per unit area per unit time per unit frequency interval:

$$N_\nu = \frac{C_4 \nu^2}{e^{K_2 \nu / T} - 1} \qquad 2.6.4$$

The constants are:

$$C_1 = 2\pi hc^2 \quad = \quad 3.7418 \times 10^{-16} \ \text{W m}^2 \qquad 2.6.5$$

$$C_2 = 2\pi c \quad = \quad 1.8837 \times 10^{9} \ \text{m s}^{-1} \qquad 2.6.6$$

$$C_3 = 2\pi h/c^2 \quad = \quad 4.6323 \times 10^{-50} \ \text{kg s} \qquad 2.6.7$$

$$C_4 = 2\pi/c^2 \quad = \quad 6.9910 \times 10^{-17} \ \text{m}^{-2} \ \text{s}^2 \qquad 2.6.8$$

$$K_1 = hc/k \quad = \quad 1.4388 \times 10^{-2} \ \text{m K} \qquad 2.6.9$$

$$K_2 = h/k \quad = \quad 4.7992 \times 10^{-11} \ \text{s K} \qquad 2.6.10$$

Symbols:
- h = Planck's constant
- k = Boltzmann's constant
- c = speed of light
- T = temperature
- λ = wavelength
- ν = frequency

2.7 *Wien's Law.*

The wavelengths or frequencies at which these functions reach a maximum, and what these maximum values are, can be found by differentiation of these functions. They do not all come to a maximum at the same wavelength. For the four functions (equations 2.6.1,2,3,4) the wavelengths or frequencies at which the maxima occur are given by:

For equation (2.6.1):

$$\lambda = W_1/T \qquad 2.7.1$$

For equation (2.6.2):

$$\lambda = W_2/T \qquad 2.7.2$$