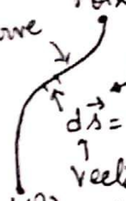
→ Dimensional variations and relations :-

✓ Vector relations for potential and field ($\vec{F}$ or $\vec{E}$ or $\vec{A}$) :- [Vector Field] or [Vector Function]
(V or ϕ).

[GRADIENT or GRAD]

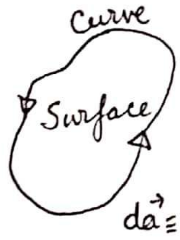


[Points Enclose Curve]

$$\therefore \phi_2 - \phi_1 = \int_{Curve} \text{grad}\,\phi \cdot d\vec{s} = \int_{Curve} \vec{\nabla}\phi \cdot d\vec{s}$$

$$\text{grad}\,\phi = \vec{\nabla}\phi = \hat{x}\frac{\partial \phi}{\partial x} + \hat{y}\frac{\partial \phi}{\partial y} + \hat{z}\frac{\partial \phi}{\partial z}$$

[STOKES]



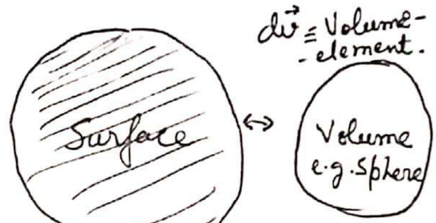
[Curve Encloses Surface]

$$\int_{Curve} \vec{A}\cdot d\vec{s} = \int_{Surface} \text{Curl}\,\vec{A}\cdot d\vec{a}$$

$$\text{Curl}\,\vec{A} = \vec{\nabla}\times\vec{A}$$

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).$$

[GAUSS].



[Surface encloses Volume].

$$\int_{surface} \vec{F}\cdot d\vec{a} = \int_{Volume} \text{div}\,\vec{F}\cdot dv$$

$$\text{div}\,\vec{F} \equiv \lim_{V_i \to 0} \int_{S_i} \vec{F}\cdot d\vec{a}_i$$

Surface of volume V_i and its surface-integral.

divergence of $\vec{F}$ - is the flux out of V_i, per unit of volume, in the limit of infinitesimal V_i.

✓ Concept of volume from surface area :- (Replacing F by E)

$$\therefore \int_S \vec{E}\cdot d\vec{a} = \sum_{i=1}^{N} \int_{S_i} \vec{E}\cdot d\vec{a}_i = \sum_{i=1}^{N} V_i\left[\frac{\int_{S_i} \vec{E}\cdot d\vec{a}_i}{V_i}\right].$$

In the limit $N \to \infty$, $V_i \to 0$ the term in the bracket becomes divergence of $\vec{E}$ and the sum goes into a volume integral, i.e

$$\int_S \vec{E}\cdot d\vec{a} = \int_V \underbrace{\text{div}\,\vec{E}}_{scalar}\cdot dv \equiv \int (\vec{\nabla}\cdot\vec{E})\cdot dv$$

→ This equation is called the Gauss Theorem or the Divergence Theorem or the differential form of the Gauss's law.

✓ If $\vec{E}$ be the d.c electric field $\therefore \int_S \vec{E}\cdot d\vec{a} = 4\pi \int_V \rho\, d\vec{v} = \int_V \text{div}\,E \cdot dv$ → This is valid for any volume i.e of any shape, size or location.

✓ So, the Gauss's law in differential form is a local relation between charge density ρ and $\vec{E}$.

Concept of Charge Density → System of point charges

↓

Charge per unit → Length (L) | Area (A) | Volume (V).

Elemental forms → dl or dL | dA | dV.

For total charge, Q → $Q/L \equiv \lambda$ | $Q/A \equiv \sigma$ | $Q/V \equiv \rho$.

↳ $Q = \lambda \times L$ $= \sigma \times A$ $= \rho \times V$ (volume)

L ↑ $(2\pi a_0)$ ↑ (for the form of ring.)

A ↑ Surface area ↑ $(4\pi a_0^2)$

V ↑ Enclosed volume ↑ $(\frac{4}{3}\pi a_0^3)$

For spherical distribution of charge, the charge density element is based on "Area" which is $dQ = \sigma\, dA$.

About Electrical field, $\vec{E}$:- (i.e $\vec{E}$ in charge free region and near vicinity of charge density $= \rho$).

1) The electric field $\vec{E}$ is a very special kind or type of vector function and not any vector function as from the Stoke's theorem $\vec{\nabla} \times \vec{E} = 0$. So, this means that if $\vec{E}$ be a vector function such that $\vec{\nabla} \times \vec{E} \neq 0$, then $\vec{E}$ is not an electrostatic field.

Q: Is $\vec{E} = y\hat{x}$ an electrostatic field?

2) Conceptually any vector whose curl is zero is equal to the gradient of some scalar fn., i.e $\vec{E} = -$ gradient of ϕ or $V = -\nabla\phi$ or $-\nabla V$ where V or ϕ is a scalar function which must be differentiable. ∴ Next, is to find $\vec{\nabla} \times \vec{\nabla}\phi$ and $\vec{\nabla} \times (-\vec{\nabla}\phi)$.

✓ Characteristics of electric field, $\vec{E}$ are (i) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ and (ii) $\vec{\nabla} \times \vec{E} = 0$.

✓ Divergence of $\vec{E}$ is the Laplacian of ϕ or V. ∴ from Gauss's law $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ → POISSON'S EQUATION.

✓ In the region of no charge, i.e. $\rho = 0$ the POISSON'S EQN reduces to LAPLACE'S EQN. i.e $\nabla^2 V = \vec{\nabla}\cdot\vec{E} = 0$.

✓ To establish $\vec{E}$ as the electric field, we need to find both the divergence $(\vec{\nabla}\cdot\vec{E})$ and the curl $(\vec{\nabla}\times\vec{E})$. Further, $\vec{\nabla}\times\vec{E} = 0$.

3) (i) For a single point charge, q at a distance $\vec{r}$; the potential is $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$.

(ii) " " collection of " "s, q_i " " " $\vec{r}_i$; " " " $\frac{1}{4\pi\varepsilon_0}\cdot\sum_{i=1}^{N}\frac{q_i}{r_i}$

(iii) " " continuous distribution charges or a volume charge $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r}\cdot dq$

$d\tau'$ is volume charge element. $= \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{r}\, d\tau'$.

4) Gauss's law in Integral form $\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enclosed}$. ↑ is the total charge enclosed within the surface. For Gauss's law in differential form, $Q_{enclosed} = \int_V \rho\cdot d\tau$.

$\int_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \int_V \left(\frac{\rho}{\varepsilon_0}\right) d\tau$ (holds for any volume) ∴ $\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$. or $\vec{\nabla}\cdot\varepsilon_0\vec{E} = \rho$ or $\vec{\nabla}\cdot\vec{D} = \rho$