What is diffraction Diffraction?

Light wave while passing through a small slit, if the observations are made carefully then one finds that if the width of the slit is not very large compared to the wavelength, then the light intensity in the geometrical dark region is not uniform and there is also some intensity inside the geometrical shadow. Further, if the width of the slit is made smaller, larger amounts of energy reach the geometrical shadow. This spreading-out of a wave when it passes through a narrow opening is usually referred to as diffraction and the intensity distribution on the screen is known as the diffraction pattern.

What is Fraunhofer and Fresnel's class of diffractions?

In the Fresnel class of diffraction the source of light and the screen are, in general, at a finite distance from the diffracting aperture. In the Fraunhofer class of diffraction, the source and the screen are at infinite distances from the aperture.

Find the intensity distribution due to a single slit.

We will first study the Fraunhofer diffraction pattern produced by an infinitely long slit of width b. A plane wave is assumed to fall normally on the slit and we wish to calculate the intensity distribution on the focal plane of the lens L [see Fig. below].

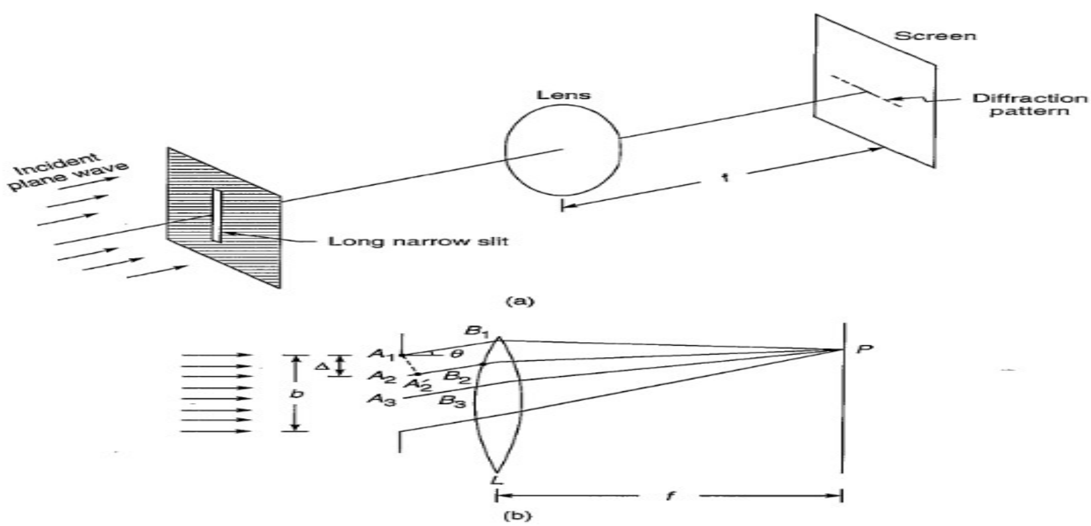

Fig- 1: Diffraction in a single slit.

We assume that the slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygens' secondary wavelets which interfere with the wavelets emanating from other points. Let the point sources be at A_1, A2, A3,... and let the distance between two consecutive points be Δ [see Fig. 1.(b)]. Thus, if the number of point sources be n, then,

$$b = (n - 1)\ \Delta \qquad \text{---------}\ (1)$$

We will now calculate the resultant field produced by these n sources at the point P, P being an arbitrary point (on the focal plane of the lens) receiving parallel rays making an angle θ with the normal to the slit [see Fig. 1.(b)]. Since the slit actually consists of a continuous distribution of sources, we will, in the final expression, let n go to infinity and Δ go to zero such that $n\Delta$ tends to b. Now, at the point P, the amplitudes of the disturbances reaching from A_1, A2)... will be very nearly the same because the point P is at a distance which is very large in comparison to b. However, because of even slightly different path lengths to the point P, the field produced by A_1 will differ in phase from the field produced by A2.

For an incident plane wave, the points A_1, A_2,... are in phase and, therefore, the additional path traversed by the disturbance emanating from the point A_2 will be A_2A_2' where A_2' is the foot of the perpendicular drawn from A_1 on $A_2\mathbf{B_2}$. This follows from the fact that the optical paths A_1B_1P and $A_2'B_2P$ are the same. If the diffracted rays make an angle θ with the normal to the slit then the path difference would be,

$A_2A_2' = \Delta \sin\theta$

The corresponding phase difference, ϕ, would be given by

$\phi = (2\pi/\lambda) \, . \, \Delta \sin\theta$ -------- (2)

Thus, if the field at the point P due to the disturbance emanating from the point A1 is a cos ωt, then the field due to the disturbance emanating from A2 would be a cos (ωt - ϕ). Now the difference in the phases of the disturbance reaching from the points A2 and A3 will also be ϕ and thus the resultant field at the point P would be given by

$E = a[\cos \omega t + \cos(\omega t - \phi) + ... + \cos(\omega t - (n-1)\phi)]$ -------- (3)

Thus,

$E = E_\theta \cos[\omega t - (1/2)(n-1)\phi]$ -------- (4)

where E_θ is given by, $E_\theta = a \,\mathrm{Sin}(n\phi/2) / \mathrm{Sin}(\phi/2)$ --------- (5)

In the limit of $n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \longrightarrow b$,

we have,

$n\phi/2 = (2\pi/\lambda)\, n \Delta \sin\theta \longrightarrow (\pi/\lambda)\, b \sin\theta$ -------- (6)

Further,

$\phi = (2\pi/\lambda)\, \Delta \sin\theta = (2\pi/\lambda)\, b \sin\theta \, . \, 1/n$ -------- (7)

would tend to zero and we may, therefore, write,

$E_\theta = na \,\mathrm{Sin}(\pi b \sin\theta/\lambda) / (\pi b \sin\theta/\lambda)$

$= A \,\mathrm{Sin}\beta / \beta$

where $\beta = (\pi b \sin\theta/\lambda)$

Thus,

$E = A \,\mathrm{Sin}\beta / \beta \,\mathrm{Cos}(\omega t - \beta)$ -------- (8)

The corresponding intensity distribution is given by

$I = I_0 \,\mathrm{Sin}^2\beta / \beta^2$

where I_0 represents the intensity at θ = 0

Found an intensity distribution due to Diffraction in Double Slit.

The Fraunhofer diffraction pattern produced by two parallel slits (each of width b) separated by a distance d. We would find that the resultant intensity distribution is a product of the single-slit diffraction pattern and the interference pattern produced by two point sources separated by a distance d.

In order to calculate the diffraction pattern we use a method similar to that used for the case of a single slit and assume that the slits consist of a large number of equally spaced point sources and that each point on the slit is a source of Huygens' secondary wavelets. Let the point sources be at Ay, A2, A3>... (in the first slit) and at By, B2, B3>... (in the second slit) [see Fig. 18.29]. As before, we assume that the distance between two consecutive points in either of the slits is A. If the diffracted rays make an angle 6 with the normal to the plane of the slits, then the path difference between the disturbances reaching the point P from two consecutive points in a slit will be A sin 6. The field produced by the first slit at the point P will, therefore, be given by [see Eq. (2)] .

$E1 = A \,\mathrm{Sin}\beta / \beta \,\mathrm{Cos}(\omega t - \beta)$

Similarly, the second slit will produce a field

$E2 = A \,\mathrm{Sin}\beta / \beta \,\mathrm{Cos}(\omega t - \beta - \phi_1)$

at the point P, where

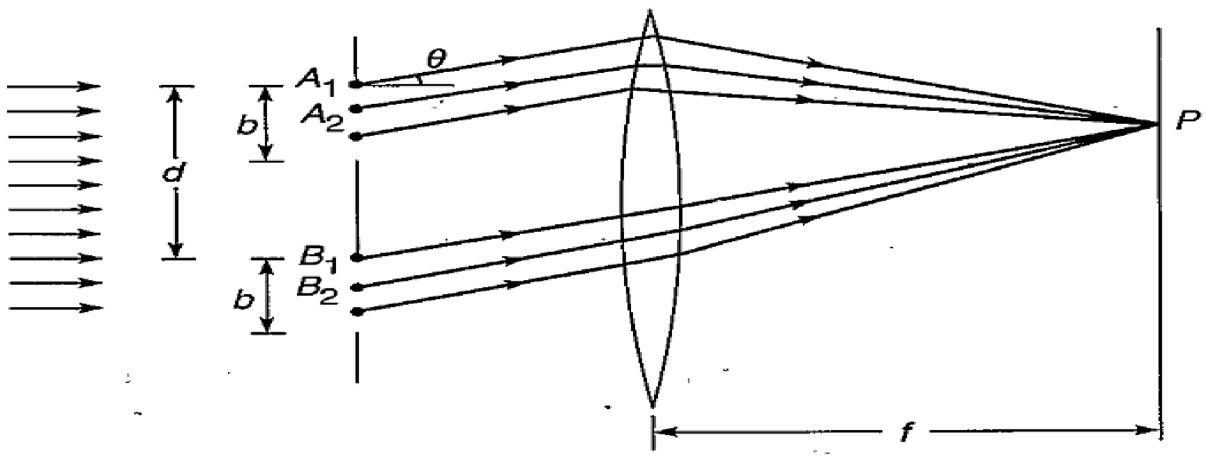

Fig.2 – Fraunhofer Diffraction in a double slit

$\phi_1 = (2\,\pi/\lambda)\, d \sin\theta$

represents the phase difference between the disturbances (reaching the point P) from two corresponding points on the slits; by corresponding points we imply pairs of points like (Ay, By), (A2, B2),... which are separated by a distance d. Hence the resultant field will be

$E = E1 + E2$

$= A \sin\beta / \beta \; [\cos (\omega t - \beta) + \cos (\omega t - \beta - \phi_1)]$

which represents the interference of two waves, each of amplitude $A \sin\beta / \beta$ and differing in phase by ϕ_1. The above equation can be rewritten in the form

$E = A \sin\beta / \beta \, . \cos \gamma \, . \cos (\omega t - ½.\beta - ½.\phi_1)]$

where $\gamma = ½.\phi_1 = (\pi/\lambda)\, d \sin\theta$

The intensity distribution will be of the form,

$$I = 4\, I_0 \; \sin^2\beta / \beta^2 \cdot \cos^2 \gamma$$

where $I_0 \; \sin^2\beta / \beta^2$ represents the intensity distribution produced by one of the slits. As can be seen, the intensity distribution is a product of two terms; the first term ($\sin^2\beta / \beta^2$) represents the diffraction pattern produced by a single slit of width b and the second term ($\cos^2 \gamma$) represents the interference pattern produced by two point sources separated by a distance d. Indeed, if the slit widths are very small (so that there is almost no variation of the $\sin^2\beta / \beta^2$ term with θ) then one simply obtains the Young's interference pattern

The intensity distribution pattern due to double slit is shown below,

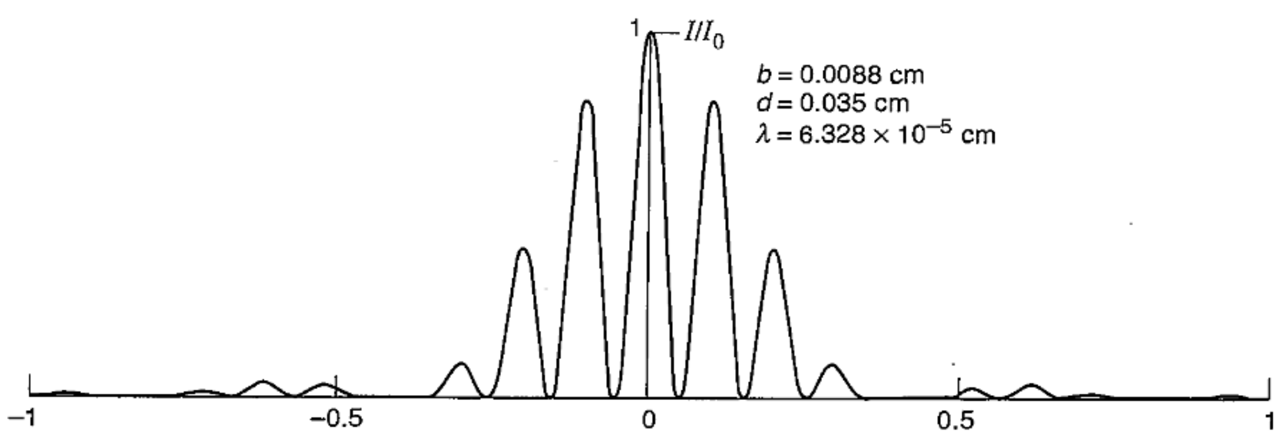

Fig.3 - Intensity distribution pattern due to double slit

Find an intensity distribution due to diffraction grating.

The diffraction pattern produced by N parallel slits, each of width b; the distance between two consecutive slits is assumed to be d. is calculated below,
As before, we assume that each slit consists of n equally spaced point sources with spacing Δ. Thus the field at an arbitrary point P will essentially be a sum of N terms:

$$E = A \,\mathrm{Sin}\beta / \beta \;[\, \mathrm{Cos}\,(\omega t - \beta) + \mathrm{Cos}\,(\omega t - \beta - \phi_1) + \mathrm{Cos}\,(\omega t - \beta - (n-1)\phi_1)]$$

where the first term represents the amplitude produced by the first slit, the second term by the second slit, etc. , the above equation can be rewritten as,

$$E = A \,\mathrm{Sin}\beta / \beta \;.\; \mathrm{Sin}\, n\gamma / \gamma \;\mathrm{Cos}[\, \omega t - \beta - (n-1)/2 \;.\; \Phi_1 \,]$$

The corresponding intensity distribution will be,

$$I = I_0\, \mathrm{Sin}^2\beta / \beta^2 \cdot \mathrm{Sin}^2 N\gamma \;/\; \mathrm{Sin}^2\gamma$$

where $I_0\, \mathrm{Sin}^2\beta / \beta^2$ represents the intensity distribution produced by a single slit. As can be seen, the intensity distribution is a product of two terms; the first term $\mathrm{Sin}^2\beta / \beta^2$ represents the diffraction pattern produced by a single slit and the second term $\mathrm{Sin}^2 N\gamma \;/\; \mathrm{Sin}^2\gamma$ represents the interference pattern produced by N equally spaced point sources.

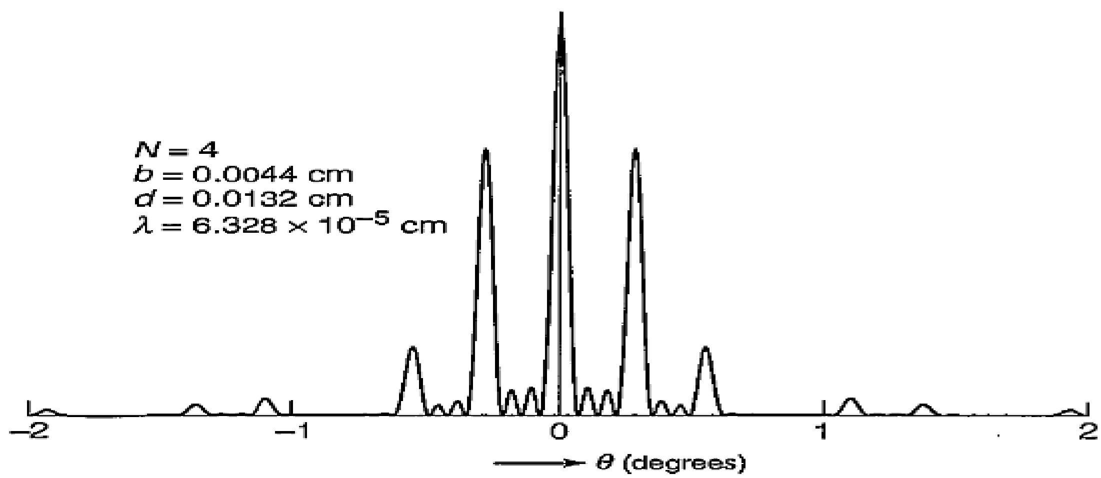

Fig. 5 - The intensity distribution corresponding to the four-slit Fraunhofer diffraction pattern as predicted by Eq. (50) corresponding to b = 0.0044 cm, d = 0.0132 cm and A, = 6.328 x 10"5 cm. The principle maxima occur at 9 = 0.275°, 0.55°, 0.82°, 1.1°, ... Notice the (almost) absent third order.

Define the resolving power of a diffraction grating and find an expression of it.

In the case of a grating the resolving power refers to the power of distinguishing two nearby spectral lines and is defined by the following equation:

$$R = \lambda / \Delta\lambda$$

where $\Delta\lambda$ is the separation of two wavelengths which the grating can just resolve; the smaller the value of $\Delta\lambda$, the larger the resolving power.
The Rayleigh criterion can again be used to define the limit of resolution. According to this criterion, if the principal maximum corresponding to the wavelength $\lambda + \Delta\lambda$ falls on the first minimum (on the either side of the principal maximum) of the wavelength λ, then the two wavelengths λ and $\lambda + \Delta\lambda$ are said to be just resolved. If this common diffraction angle is represented

by θ and if we are looking at the mth order spectrum, then the two wavelengths λ and $\lambda + \Delta\lambda$ will be just resolved if the following two equations are simultaneously satisfied:

$$d \sin\theta = m(\lambda + \Delta\lambda)$$

and $$d \sin\theta = m\lambda + \lambda/N$$

Thus,

$$R = \lambda / \Delta\lambda = m.N$$

which implies that the resolving power depends on the total number of lines in the grating—obviously on only those lines which are exposed to the incident beam. Further, the resolving power is proportional to the order of the spectrum.

References :

1) Optics – Ajoy Ghatak, download link